

# Linear Programming

- ① Assign variables
- ② Write Objective Function
- ③ Write Inequalities
- ④ Isolate  $y$ ; Graph ([desmos.com](https://www.desmos.com))
- ⑤ Identify corner points
- ⑥ Test corner points in objective function to find max or min
- ⑦ Express answer as a sentence

A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for \$1.10 per litre. Heating oil is projected to sell for \$1.75 per litre. The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

① Assign variables

$x =$  litres of gas

$y =$  litres of oil

② Objective Function

$$R = 1.10x + 1.75y$$

$$V = 0 + 0 = 0 \text{ (min)}$$

$$V = 0 + 4 = 4$$

$$V = 6 + 4 = 10$$

$$V = 12 + 0 = 12 \text{ (max)}$$

③ Write Inequalities

$$y \leq 9000000$$

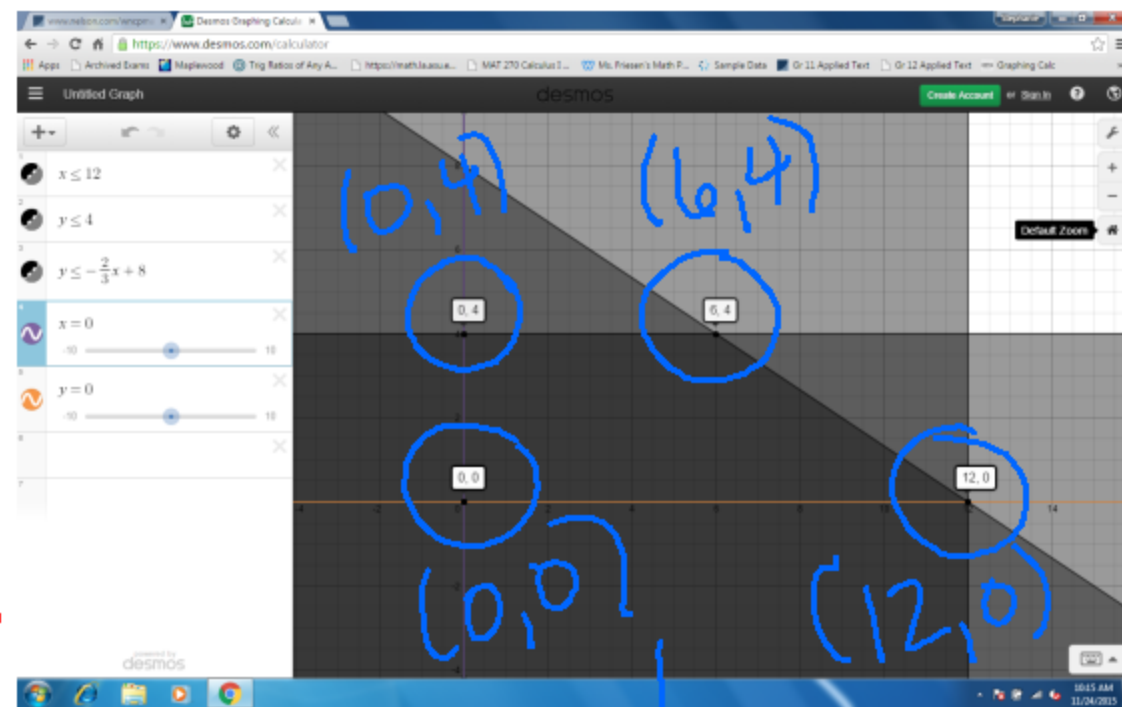
$$x \leq 6000000$$

$$x \geq 2y$$

$$x \geq 0$$

$$y \geq 0$$

$$y \leq -\frac{2}{3}x + 8$$



use as  
 $x$  and  
 $y$  min  
in  
window



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- ① Assign variables  
 $x =$  litres of gas  
 $y =$  litres of oil
- ② Objective Function  
 $R = 1.10x + 1.75y$

③ Write Inequalities

$$y \leq 9\,000\,000$$
$$x \leq 6\,000\,000$$

$$x \geq 2y$$

$$x \geq 0 \quad (x=0)$$

$$y \geq 0 \quad (y=0)$$

④ Isolate  $y$ :

$$\frac{x}{2} \geq \frac{2y}{2}$$

$$y \leq \frac{x}{2}$$

⑤ Graph

See end of notes for graph

⑥ Use objective function

$$R = 1.10(0) + 1.75(0)$$
$$= 0$$

$$R = 1.10(6\,000\,000) + 1.75(3\,000\,000)$$

max  $= \$11\,850\,000$

$$R = 1.10(6\,000\,000) + 1.75(0)$$
$$= 6\,600\,000$$

**EXAMPLE 2** Creating a model for an optimization problem, and solving the problem

L&G Construction is competing for a contract to build a fence.

- The fence will be no longer than 50 yd and will consist of narrow boards that are 6 in. wide and wide boards that are 8 in. wide.
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost \$3.56 each, and the wide boards cost \$4.36 each.

Determine the maximum and minimum costs for the lumber to build the fence.



① Assign variables  
 $x =$  narrow boards  
 $y =$  wide boards

② Objective Function

$$C = 3.56x + 4.36y$$

③ Inequalities

$$1 \text{ yd} = 36 \text{ in}$$

$$50(36) = 1800 \text{ in.}$$

$$6x + 8y \leq 1800$$

$$y \geq 100$$

$$x \leq 80$$

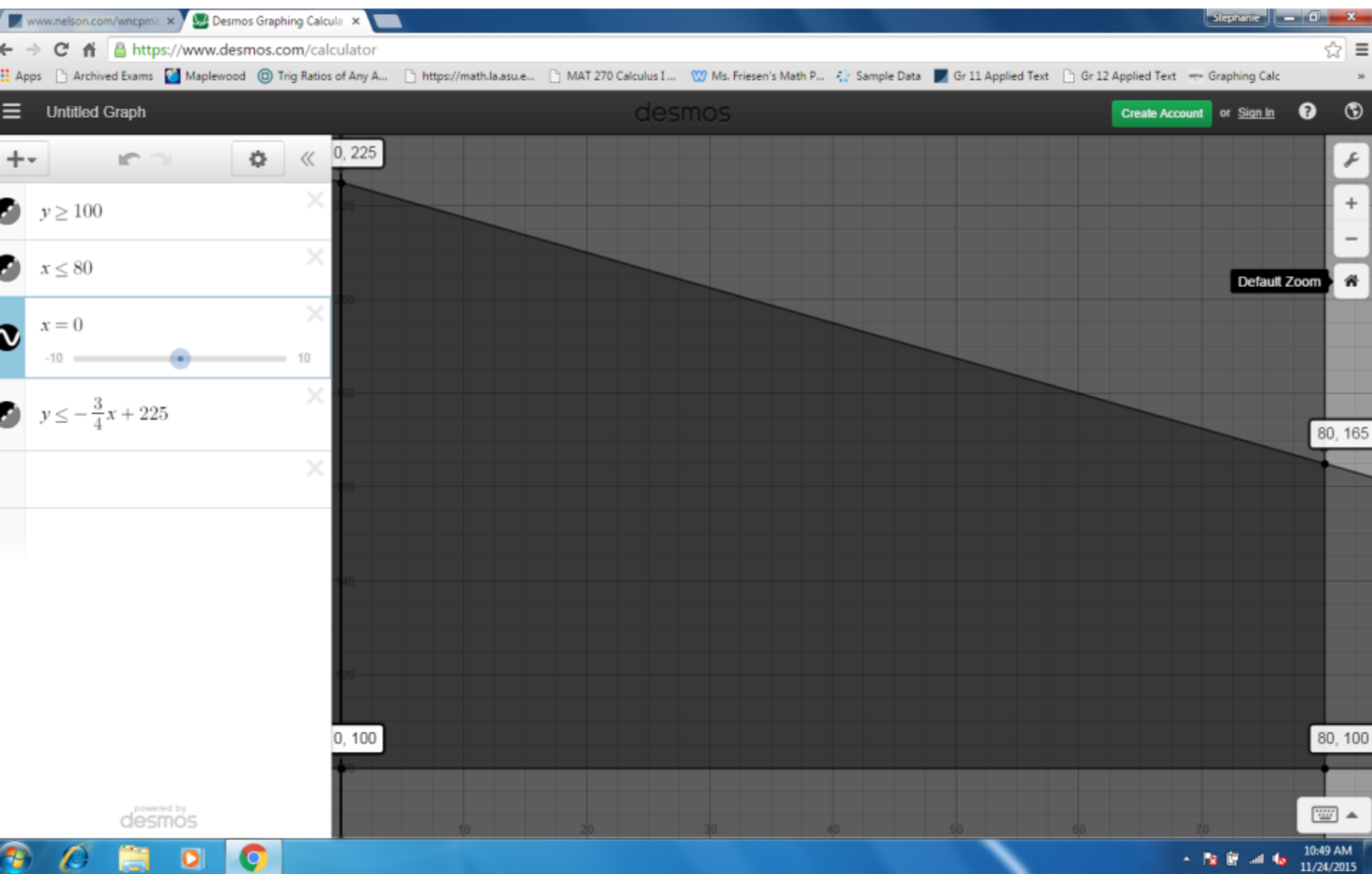
$$x \geq 0 \quad (x = 0)$$

Isolate  $y$

$$8y \leq -6x + 1800$$

$$y \leq -\frac{6x}{8} + \frac{1800}{8}$$

$$y \leq -\frac{3}{4}x + 225$$



0 narrow boards and 100 wide would minimize cost. 80 narrow boards and 165 wide boards would maximize cost.

$$C = 3.56x + 4.36y$$

$$(0, 100)$$

$$C = 3.56(0) + 4.36(100)$$

$$= \$436$$

$$(0, 225)$$

$$C = 3.56(0) + 4.36(225)$$

$$= \$981$$

$$(80, 165)$$

$$C = 3.56(80) + 4.36(165)$$

$$= \$1004.20$$

$$(80, 100)$$

$$C = 3.56(80) + 4.36(100)$$

$$= \$720.90$$

1  $x = 0$

2  $y = 0$

3  $y \leq 9000000$

4  $x \leq 6000000$

5  $y \leq \frac{x}{2}$

6

7

