Optimization Problems

optimization problem: a problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

constraint: a limiting condition of the optimization problem being modelled, represented by a linear inequality.

objective function: In an optimization problem, the equation that represents the relationship between the 2 variables in the system of linear inequalities and the quantity to be optimized.

feasible region: the solution region for a system of linear inequalities

Ex. 1:

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

Objective Function:
(Optimization Equation)
*Trying to minimize & maximize Cost.

① Assign variables
Racing cars → x
SUVs → y

② Write eqn.
Cost = 8x + 12y

③ Write Inequalities
$x \leq 40$
$y \leq 60$
$x + y \geq 70$

*Implicit Inequalities
$x \geq 0$
$y \geq 0$

Ex.2:

Three teams are travelling to a basketball tournament in cars and minivans.

- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles.

Create a model to represent this situation.

① Assign variables

x = # of cars

y = # of minivans

② Write objective function

$V = x + y$

③ Write Inequalities

$x \leq 12$

$y \leq 4$

$4x + 6y \leq 48$

# of people: $3(16) = 48$

$x \geq 0$

$y \geq 0$

A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for $1.10 per litre. Heating oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

① Assign variables
x = litres of gas
y = litres of oil

② Objective Function
$R = 1.10x + 1.75y$

③ Write Inequalities

$y \leq 9\,000\,000$

$x \leq 6\,000\,000$

$x \geq 2y$

$x \geq 0$

$y \geq 0$

HW: p. 330-331
#1-7 (choose 4)